Universidade Federal do Rio de Janeiro
Graduate program in Mathematics

Sector: Applied Mathematics
Research group: Complexity and Foundations of Computational Mathematics.

1. Outline

Algorithms are mathematical objects. The development of *proven* efficient algorithms is a challenge that requires tools from several areas of mathematics.

Complexity is the study of lower or upper bounds for the efficiency of *all* algorithms for a certain problem. Finding precise lower bounds can be extremely difficult. (See here how to earn a million dollars with a single one).

We are particularly interested in algorithms and complexity for continuous problems, as such that arise in numerical analysis.

Impact and applications. From a technological viewpoint, a better mathematical understanding of numerical analysis means faster and more reliable algorithms.

In particular, we do not have a satisfactory technology to solve systems of polynomial equations. Improvemnents in this technology would be useful for subjects as mechanical engineering, chemical/biochemical kinetics, computer graphics, nonlinear optimization, control theory, etc...

Connections with other subjects within mathematics. Input and output spaces of numerical problems may be linear spaces or more generally smooth manifolds. One may assume a probability measure in input space and also some group action invariance.

Invariants as the *condition number* can be then interpreted as a random variable, and the probability for a problem to be ill-posed can be estimated. But the condition number can also be related with the reciprocal distance to a discriminant variety, and may be bounded in terms of the arithmetic properties of the discriminant.

2. Participants

Professor:	Gregorio Malajovich	PhD, Berkeley.
Estudantes:	Felipe Bottega Diniz	MSc student
	Yuri da Silva Villas Boas	Undergraduate student
	Douglas Luizeto	Undergraduate student

3. Research projects

Project	**Coordinator**	**Page**
Complexity of Numerical Algorithms	Gregorio Malajovich	See project (Port)
International Cooperation(MathAmSud)	Gregorio Malajovich	See project (eng)

4. Recent publications ($\geq$ 2007) and preprints

2007

[1] Vinicius Gripp Barros Ramos, *Curvas Algébricas e Geometria Tropical*, Dissertação de Mestrado, Universidade Federal do Rio de Janeiro, Julho de 2007, http://teses.ufrj.br/IM_M/ViniciusGrippBarrosRamos.pdf.

[2] Gregorio Malajovich and Klaus Meer, *Computing minimal multi-homogeneous Bézout numbers is hard*, Theory Comput. Syst. **40** (2007), no. 4, 553–570, available at http://dx.doi.org/doi:10.1007/s00224-006-1322-y.

2008

[3] Felipe Cucker, Teresa Krick, Gregorio Malajovich, and Mario Wschebor, *A numerical algorithm for zero counting I: Complexity and accuracy*, Journal of Complexity **24** (2008), no. 5-6, 582-605, DOI 10.1016/j.jco.2008.03.001.

[4] Jean-Pierre Dedieu and Gregorio Malajovich, *On the number of minima of a random polynomial*, J. Complexity **24** (2008), no. 2, 89–108, available at http://dx.doi.org/doi:10.1016/j.jco.2007.09.003.

[5] Gregorio Malajovich, *Geometria de Algoritmos Numéricos*, Notas em Matemática Aplicada, vol. 36, SBMAC, São Carlos, setembro de 2008. http://www.labma.ufrj.br/~gregorio.

2009

[6] Felipe Cucker, Teresa Krick, Gregorio Malajovich, and Mario Wschebor, *A numerical algorithm for zero counting II: Distance to Ill-posedness and Smoothed Analysis*, Journal of Fixed Point Theory and Applications **6** (2009), no. 2, 285-294, DOI 10.1007/s11784-009-0127-4.

2010

[7] Carlos Beltrán, Jean-Pierre Dedieu, Gregorio Malajovich, and Mike Shub, *Convexity properties of the condition number*, SIAM Journal on Matrix Analysis and Applications **31** (2010), no. 3, 1491-1506, DOI 10.1137/080718681.

[8] Caio Guimarães Souza, *Estimativas sobre a convergência da iteração de Graeffe tangente*, Dissertação de Mestrado, Universidade Federal do Rio de Janeiro, Agosto de 2010, http://www.pg.im.ufrj.br/teses/Matematica/Mestrado/307.pdf.

2011

[9] Gregorio Malajovich, *Nonlinear Equations*, Publicações de Matemática, 28ž Colóquio Brasileiro de Matemática, IMPA, Rio de Janeiro, 2011.

$\geq$ 2012

[10] Felipe Cucker, Teresa Krick, Gregorio Malajovich, and Mario Wschebor, *A numerical algorithm for zero counting III: Randomization and Condition*, Vol. 48, 2012.

[11] Carlos Beltrán, Jean-Pierre Dedieu, Gregorio Malajovich, and Mike Shub, *Convexity properties of the condition number*, SIAM Journal on Matrix Analysis and Applications **33** (in print), no. 3, 905–939, available at http://dx.doi.org/doi:10.1137/100808885.

[12] Jean-Pierre Dedieu, Gregorio Malajovich, and Michael Shub, *Adaptative Step Size Selection for Homotopy Methods to Solve Polynomial Equations*, IMA Journal of Numerical Analysis **33** (2013), 1–29.

[13] Gregorio Malajovich, *On the expected number of zeros of nonlinear equations*, available at http://arxiv.org/abs/1106.6014. Preprint, ArXiV, Jul 2011.

5. Recommended curriculum

Boldface courses are mandatory for all students. Other requirements appear within parenthesis.

MSc	
Linear Algebra MAE 709 **(English proficiency exam)**	Advanced Calculus I MAE709
Advanced Calculus II MAE702 **(Qualification exam)**	**Advanced Calculus III** MAE703
Algebra 1 MAE733 Numerical Linear Algebra MAE733	Numerical Analysis MAE721 **(Math colloquium 1)**
(Math colloquium 2) **(Dissertation)**	**(Teaching stage 1)**
Doctorate	
Complexity of Numerical Algorithms MAE8xx **(English proficiency exam)**	Riemannian geometry MAC855
Algebraic Geometry 1 MAA876	Free choice
Algebraic number theory MAA875 **(Second language proficiency exam)**	Free choice
(Teaching stage 1)	(Individual research)
(Teaching stage 2)	(Individual research)
(Individual research)	
(Individual research)	**(Thesis)**

Notes:

(1) Advanced students may replace Advanced Calculus I by Topological Methods or Differential Geometry, but only after taking the other advanced calculus courses.

(2) Doctoral students with an MSc outside our program must still enroll in two more MSc-level or PhD-level courses.
(3) Doctoral courses do not need to follow the order.

6. Suggested qualifying exam program

- Numerical Linear Algebra, plus a specific course such as Complexity of Numerical Algorithms.
- If this is the main topic, the student is required to read three current papers selected with the advisor.
- Also, if this is the main topic, the candidate chooses a secondary topic.

7. More information on the subject

Foundations of Computational Mathematics is a scientific association that connects reserchers in this topics. It promotes a major confernce every three years, thematic programs, and owns the journal with the same name. (Springer Verlag). Another important journal in the subject is the Journal of Complexity.

A general introduction to the complexity of algorithms may be found in [14] (Graduate level. Requires algebra and differential geometry). For algebraic complexity, the reference is [15].

For the complexity of nonlinear equation, see [9].

A classical treaty exploring the connections between computing and mathematics is [16].

An introduction to classical computer science is [17] (Undergraduate level. Features NP-completeness.)

Further references

[14] Lenore Blum, Felipe Cucker, Michael Shub, and Steve Smale, *Complexity and real computation*, Springer-Verlag, New York, 1998.
[15] Peter Bürgisser, Michael Clausen, and M. Amin Shokrollahi, *Algebraic complexity theory*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 315, Springer-Verlag, Berlin, 1997. With the collaboration of Thomas Lickteig.
[16] Donald E. Knuth, *The art of computer programming. Vol. 2*, 2nd ed., Addison-Wesley Publishing Co., Reading, Mass., 1981. Seminumerical algorithms; Addison-Wesley Series in Computer Science and Information Processing.
[17] Michael R. Garey and David S. Johnson, *Computers and intractability*, W. H. Freeman and Co., San Francisco, Calif., 1979. A guide to the theory of NP-completeness; A Series of Books in the Mathematical Sciences.